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ONE WAY TO A PRIORI EVALUATION OF THE UNCERTAINTY OF MEASURING INSTRUMENTS DURING ITS OPERATION IN A DYNAMIC MODE

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Анотація

Запропоновано спосіб апріорної оцінки динамічної невизначеності вимірювальних приладів на основі математичної моделі частотної характеристики вимірювального приладу та моделі спектральної функції вхідного сигналу. Отримано модельне рівняння для оцінки амплітудного значення динамічної невизначеності вимірювань, яке зумовлено обмеженими властивостями вимірювальних пристроїв, коли вимірювальний сигнал проходить через нього в динамічних режимах роботи.

Ключові слова: динамічні вимірювання, невизначеність, частотна характеристика, спектральна функція, невизначеність типу B.

Abstract

A method for estimating the dynamic uncertainty of measuring instruments based on a mathematical model of the frequency characteristic of a measuring instrument and a model of the spectral function of an input signal is presented. The model equation for estimating the amplitude value of the dynamic measurement uncertainty is obtained, which is caused by the limited properties of the measuring devices when a measuring signal passes through it in dynamic operation modes.

Keywords: dynamic measurements, uncertainty, frequency characteristic, spectral function, uncertainty of type B.

INTRODUCTION

When compiling a report on the results of dynamic measurements, it is necessary to demonstrate quantitative values of the quality of measurements so that their reliability can be correctly assessed [1 - 6]. Without such values, the results of dynamic measurements cannot be compared, neither with each other nor with reference values. Therefore, it is necessary to propose methods for estimating the quality characteristics of dynamic measurements. In this case, it is necessary to take into account the fact that during dynamic measurements a transient mode of operation of the measuring devices (MD) will also be present at some stage, during which the signal from the output of the measuring device changes significantly over time. These circumstances are due to the inertial properties of the MD, since they consist, as a rule, of a set of different masses and springs, capacitances and inductances, and other inertial elements that lead to the manifestation of dynamic uncertainty. The equation of the transformation of the MD, which displays its statics properties, is unacceptable in a dynamic mode. In this case, we must go to the differential equations that describe the dynamic relationship between the output y(t) and the input x(t) values of the measuring devices [3 - 8].

In view of the above, there is a need to develop methods for estimating the uncertainty of dynamic measurements that would meet international requirements for estimating the characteristics of the quality of measurements, which is a topical scientific task in the field of metrology.

MAIN MATERIALS OF THE RESEARCH

The dispersion of the output signal σ_Y^2 for dynamic measurements can be defined as the square root of the integral of the spectral density of the output signal over all frequencies

$$\sigma_Y = \pi^{-1/2}\left(\int_0^{\infty} H_Y(\omega)d\omega\right)^{1/2}. \qquad (1)$$

The dispersion of the output signal σ_{Y}^2 (1) can be represented on the basis of the spectral density of the input signal and the frequency characteristic of the measuring channel used (measuring means)

$$\sigma_{\mathrm{Y}} = \pi^{-1/2}\left(\mathrm{T}^{-1}\int_0^\infty \left|K_C(j\omega)\right|^2 \left|X(j\omega)\right|^2 d\omega\right)^{1/2}, \tag{2}$$

where $\left|\mathrm{K}_{\mathrm{C}}(\mathrm{j}\omega)\right|$ is the frequency response module of the measuring device, used for dynamic measurements; $X(j\omega)$ is the spectral function of the incoming signal is related to its time function $X(t)$ by the Laplace expression [8 - 14].

Similarly, the variance of the error signal, reduced to the input, will be equal to

$$\sigma_{\Delta X} = \pi^{-1/2}\left(\int_0^\infty \left|\frac{K_C(j\omega)}{K_C}-1\right|^2 \left|X(j\omega)\right|^2 d\omega\right)^{1/2}. \tag{3}$$

The relationship between the images of the output and input values gives us an expression for the transfer function of the measuring device

$$\mathrm{K}_{\mathrm{C}}(s) = Y(s)/X(s) = \sum_{k=0}^{m} B_k s^k \Big/ \sum_{q=0}^{n} A_q s^q, \tag{4}$$

where $Y(s)$, $X(s)$ are the operator images of $Y(t)$ output and $X(t)$ input signals, respectively; k, q are the order of the derivatives of Y and X, respectively; A_q, B_k are the coefficients of the differential equation [12, 15].

Therefore, we can write that

$$H_Y(\omega) = \left|K_C(j\omega)\right|^2 H_X(\omega), \tag{5}$$

where $K_C(j\omega)$ is the frequency characteristic of the measuring transducer [4, 15].

The spectral density of the error signal, reduced to the input, will be equal to

$$\mathrm{H}_{\Delta\mathrm{X}}(\omega) = \left|\frac{K_C(j\omega)}{K_C}-1\right|^2 H_X(\omega), \tag{6}$$

K_{C} - frequency characteristic of the measuring device for $\omega = 0$.

The dynamic error of the measuring transducer, brought to the input, in the time domain can be represented by the expression

$$\Delta X(\mathrm{t}) = \pi^{-1/2}\int_0^\infty \sigma_{\Delta\mathrm{X}} e^{j\omega t} d\omega = \pi^{-1/2}\left[\int_0^\infty \sigma_{\Delta\mathrm{X}}\cos(\omega\mathrm{t})d\omega + j\int_0^\infty \sigma_{\Delta\mathrm{X}}\sin(\omega\mathrm{t})d\omega\right]. \tag{7}$$

Thus, the dynamic error that is introduced due to the limited properties of the measuring device used for the dynamic measurements can be estimated in the time domain, based on the model equation of the spectral function of the input signal and the frequency response of the measuring instrument used by formula (7).

Since expression (7) consists of real and imaginary parts, and in assessing the error we are interested in the amplitude value of dynamic error, expression (7) may now be written as

$$\left|\Delta X(\mathrm{t})\right| = \left[\left(\int_0^\infty \pi^{-1/2}\sigma_{\Delta\mathrm{X}}\cos(\omega\mathrm{t})d\omega\right)^2 + \left(\int_0^\infty \pi^{-1/2}\sigma_{\Delta\mathrm{X}}\sin(\omega\mathrm{t})d\omega\right)^2\right]^{1/2}. \tag{8}$$

When, for instance, prior knowledge about the amplitude of the frequency spectrum of the measurand is available in terms of a frequency function, then equation (8) can be rewritten in the form

$$\left|\Delta X(\mathrm{t})\right| = \left[\left(\int_0^{\pi\mathrm{F}_s} \pi^{-1/2}\sigma_{\Delta\mathrm{X}}\cos(\omega\mathrm{t})d\omega\right)^2 + \left(\int_0^{\pi\mathrm{F}_s} \pi^{-1/2}\sigma_{\Delta\mathrm{X}}\sin(\omega\mathrm{t})d\omega\right)^2\right]^{1/2}, \tag{9}$$

with $\mathrm{F}_s=1/\mathrm{T}_s$ the sampling frequency.

According to GUM Supplement 2 [1], the uncertainty associated with the estimation error is then given as the variance of the corresponding rectangular probability distribution, i.e. the dynamic uncertainty of the measuring device can be estimated from the formula

$$u_D(t)=\sqrt{\left(\int_0^{\pi F_s}\pi^{-1/2}\sigma_{\Delta X}\cos(\omega t)d\omega\right)^2+\left(\int_0^{\pi F_s}\pi^{-1/2}\sigma_{\Delta X}\sin(\omega t)d\omega\right)^2}\Bigg/\sqrt{3}\,. \qquad (10)$$

CONCLUSIONS

The proposed spectral method of evaluating the uncertainty of dynamic measurements allows the calculation of the amplitude values of dynamic uncertainties, taking into account the international requirements for the evaluation of the quality of measurements - the concept of uncertainty. It helps to ensure the uniformity of measurements and enables comparison of the results of dynamic measurements made by different measuring devices and testing by different laboratories of leading countries.

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